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ABSTRACT

The role of visual-spatial operations in nonroutine mathematical problem solving in schoolchildren is investigated through a search of related literature. It is suggested that certain visual-spatial operations are concomitant with the mental processes necessary in problem solving. The following topics are discussed: (1) the importance of problem solving; (2) the process of problem solving; (3) visual-spatial factors and mental processes; (4) the visual-spatial approach to problem solving; (5) visual-spatial skills and mathematics achievement; and (6) visual-spatial skills and sex. (MP)

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VISUAL-SPATIAL OPERATIONS AND THE DEVELOPMENT OF PROBLEM SOLVING ABILITIES

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INTRODUCTION

This paper investigates the role of visual-spatial operations in nonroutine mathematical problem solving in schoolchildren. It will be suggested that certain visual-spatial operations are concomitant with the mental processes necessary in problem solving.

The conceptual framework developed with John C. Moyer of Marquette University in large part is an outgrowth of involvements in the Georgia Center for the Study of Learning and Teaching Mathematics. In particular, it is influenced by cooperative research of the Space and Geometry Working Group of the Georgia Center resulting in a monograph (Lesh and Mierkiewicz, 1977).

THE LITERATURE

Importance of Problem Solving

Three major basic skills in mathematics education are often cited: (1) the formation of mathematical concepts, (2) the acquisition of computational skills, and (3) the development of problem solving abilities. Furthermore, it is said that problem solving is the principle reason for the study of mathematics. Several models of learning place problem solving, which is also referred to as the nonroutine application of concepts and skills, in a very high position in the cognitive scheme (Avital and Shettleworth, 1968; Bloom, 1956, Brownell and Hendrickson, 1950; Gagne',

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1970; Romberg and Wilson, 1968). Current literature (Kantowski, 1977; Krutetskii, 1976; LeBlanc, 1977; Lester, 1977; Robinson, 1977; Suydam and Weaver, 1977; Wheatley, 1977) demonstrates a continuing interest in the development of problem solving skills in the young learner.

It is not uncommon for problem solving to be the downfall of the mathematics student. The ability to analyze, synthesize, and generate original schemes for solution is quite different from the application of routine procedures often required in the mathematics class. It should be mentioned that problem solving skills are not to be associated exclusively with word problems. Here problem solving includes the more important process of dealing with a new situation or task in which previously learned concepts and skills need to be applied (Gagne', 1970). Avital and Shettleworth (1968) refer to this process as an "open search," which requires a re-arrangement and restatement of the given information, leading to new relationships and thus solution.

The Process of Problem Solving

Kantowski (1977) describes a recent shift in research interests from the product aspect of problem solving to that of process. The November 1977 issue of the Arithmetic Teacher is devoted to the topic of problem solving and demonstrates a concern for process over product in research in the three fields of mathematics education, psychology, and neurology. However, Lester (1977) points out that "due to the complex nature of problem solving, there is little universally accepted knowledge about the best ways to enhance children's problem solving abilities" (p. 12).

Thus, although process is to be stressed, research sheds little light on possible methods for such emphasis in the schools. He further adds that psychological research findings are an extremely welcome contribution to mathematics education research.

Visual-Spatial Factors and Mental Processes

French (1965), Michael, Guilford, Fruchter, and Zimmerman (1957) and Michael, Zimmerman, and Guilford (1951) are among the researchers who have examined the relationship between visual-spatial factors and mental processes. Results are mixed, due to at least one factor, that of imperfectly refined techniques for such investigations. Brinkman (1966) found a way to improve spatial visualization through programmed instruction, but suggested that increased visual ability might increase learning in content areas.

Neurophysiology has made significant contributions to mathematics education research, supporting the colloborative roles of visual-spatial skills in mental processes (Dilling, Wheatley, and Mitchell, 1976; Galin and Ornstein, 1972; Gazzaniga, 1970; Levy, Trevarthen, and Sperry, 1972; McGlone and Davidson, 1973). These findings help to provide a more complete picture of the cognitive development which Piaget and Inhelder (1969) described in The Psychology of the Child. Wheatley, Frankland, and Mitchell (1978) point out that neurology can now be used to describe the stages of cognitive development. In addition, it is now known that there are two methods for processing information: one is verbal, linear, and analytical; the other is nonverbal, spatial, and experiential. Experimenters in neurology, psychology, and mathematics education are now combining efforts to understand the mental processes and visual-spatial skills used in problem solving.

The Visual-Spatial Approach to Problem Solving

The following compilation of skills which are required for successful problem solving is derived from the research findings of psychology and mathematics education. Although the original authors made no direct mention of visual-spatial skills, the listed skills seem to include visual-spatial components, as is indicated in parentheses.

(1) Recognizing the salient features of the problem solving situation (Simon, 1976); (figure-ground discrimination, i.e., bringing the most important aspects into focus).

(2) Constructing representatives of the problem situation (Simon, 1976); (figure-ground discrimination and change of perspective, i.e., bringing details into focus and re-examining the situation from the newly-specified point of view).

(3) Perceiving and then overcoming surmountable obstacles (Maier, 1970); (change of perspective, i.e., shifting mental position in order to see the obstacle).

(4) Generating alternative ideas (Maier, 1970); (figure-ground discrimination, i.e., bringing various aspects of the situation into focus in succession).

(5) Understanding of appropriate relationships (Brownell, 1972); (pattern recognition, i.e., perceiving similarities differences, causes, and effects).

Suydam and Weaver (1977) report the following helpful strategies or techniques for problem solving. Again, the visual-spatial components of the skills are described parenthetically.

(1) Dramatize problem situations and solutions; (change of perspective, i.e., mentally move from disinterested observer

to active participant).

(2) Make drawings and diagrams; (figure-ground discrimination, i.e., focus on the most important aspects and eliminate unnecessary words).

(3) Work without numbers; (figure-ground discrimination, i.e., focus on relationships rather than particular numbers).

(4) Note likenesses, differences, and analogies; (pattern recognition, i.e., perceive similarities, differences, and relationships).

Although there is a growing tendency to provide students with exercises involving visual-spatial skills and flexible mental imagery, very little appears to be known about these skills and their specific role in the mental exercises of problem solving.

Arnheim (1969) relates perception, problem solving, and visual-spatial abilities in the following paragraph.

> My contention is that the cognitive operations called thinking are not the privilege of mental processes above and beyond perception but the essential ingredients of perception itself. I am referring to such operations as active exploration, selection, grasping of essentials, simplification, abstraction, analysis and synthesis, completion, correction, comparison, problem solving, as well as combining, separating, putting into context. (p. 13)

The term "cognitive," Arnheim explains, implies all the mental processes of sensory perception, memory, thinking and learning. The applications of sensory perception are visual-spatial operations as outlined by McKim (1972). They are (1) closure as in figure-ground discrimination of rigid configurations, (2) pattern recognition of rigid configurations, (3) pattern completion of a

drawn figure, (4) mental transformation of a configuration in space through a rotation or reflection as in changing perspective, (5) orthographic projection by imagining different visual perspectives of a solid or by imagining outcomes of actions or dynamic structures such as with paper folding patterns or string knots.

Visual-Spatial Skills and Mathematics Achievements

Recent work by Guay and McDaniel (1977) and Schultz (1978, 1976) has shown a correlation between mathematics achievement and spatial skills. Discrimination of embedded figures in rigid configurations was found by Moyer (1978) to be significantly related to developmental level. However, there is little evidence of research looking specifically at the role of visual-spatial skills in problem solving (Werdelin, 1961). In a summary of this area, Fennema (1974) states that "the relationship between learning in mathematics and spatial ability is not clear and the need for more data is great" (p. 7).

Visual-Spatial Skills and Sex

The literature is neither consistent nor strong on the role of sex in visual-spatial skills and problem solving abilities. Recently, Fennema (1974) and Maccoby and Jacklin (1973) found in the literature that male superiority, although often of only a small degree, is usually indicated. The problem solving abilities of boys and girls have been investigated, with contradictory results. Some (Sheehan, 1968) indicate male superiority; others (Cleveland and Bosworth, 1967) indicate no differences between males and females.

CONCLUSION

It is an accepted fact that the emphasis in mathematics education is to do away with the "crutches" for learning and proceed on to algorithms and answers as soon as possible. This, of course, means a fast weaning of the nonverbal visual learning experiences, which are at best precious little as it is. Consider the implications of this. Perhaps current mathematics curricula leave out the most crucial element of mathematics learning. Nonverbal visual perception is seen by many (Ornstein, 1972; McKim, 1972; Arnheim, 1969; Samples, 1977; Wheatley, 1977) as an area of human experience that is sadly neglected in our educational system and therefore needs to be considered in future research and instruction.

REFERENCES

Arnheim, R. Visual Thinking. Berkeley, Calif.: Univ. of Calif. Press, 1969.

Avital, S. H., and Shettleworth, S. J. Objectives for Mathematics Learning - Some Ideas for the Teachers. Toronto: Ontario Institute for Studies in Educational Bulletin, 1968, No. 3.

Bloom, B. S., ed., Taxonomy of Educational Objectives: The Classification of Education Goals. Handbook 1: Cognitive Domain. New York: David McKay Co., 1956.

Brinkman, E. H. "Programmed Instruction as a Technique for Improving Spatial Visualization." Journal of Applied Psychology, 1966, 50, 179-184.

Brownell, W. A. "Problem Solving." In The Place of Meaning in Mathematics Instruction: Selected Theoretical Papers of William A. Brownell, edited by Weaver, J. F., and Kilpatrick, J. Studies in Mathematics series. 21, 129-159. Stanford, Calif.: School Mathematics Study Group, 1972.

Brownell, W. A. and Hendrickson, G. "How Children Learn Information, Concepts and Generalizations," in Learning and Instruction, Forty-ninth Yearbook of The National Society for The Study of Education, pt., 1, Henry, N. B. (ed), Chicago: University of Chicago Press, 1950, 92-120.

Cleveland, G. A. and Bosworth, D. L. "A Study of Certain Psychological and Sociological Characteristics as Related to Arithmetic Achievement." Arithmetic Teacher, 1967, 14, 387.

Dilling, R., Wheatley, G., and Mitchell, R. "An EEG Investigation of the Differences in the Hemispheric Specialization of Formal and Concrete Operational Persons." Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, California, April 1976.

Fennema, E. "Mathematics, Spatial Ability and the Sexes." Paper presented at the Annual Meeting of the American Educational Research Association. Chicago, Illinois, April 1974.

French, J. W. "The Relationship of Problem-Solving Styles to the Factor Composition of Tests." Educational and Psychological Measurement, 1965, 25, 9-27.

Gagne', R. The Conditions of Learning, New York: Holt, Rinehart & Winston, 1970.

Galin, D. and Ornestein, R. "Lateral Specialization of Cognitive Mode: An EEG Study." Psychophysiology, 9, 1972, 412-418.

Gazzaniga, M. Cerebral dominance viewed as a decision system. In S. Dimond and J. Beaumont (eds.) Hemispheric Function in the Human Brain. New York: John Wiley & Sons, 1970.

Guay, R. B. and McDaniel, E. D. "The Relationship between Mathematics Achievement and Spatial Abilities Among Elementary School Children." Journal for Research in Mathematics Education, 1977, 8, 211-215.

Kantowski, M. G. "Processes Involved in Mathematical Problem Solving." Journal for Research in Mathematics Education, 1977, 8, 163-180.

Krutetskii, V. A. The Psychology of Mathematical Abilities in School Children. Translated by Teller, J.; Kilpatrick, J., and Wirszup, I., (eds.) University of Chicago Press, 1976.

LeBlanc, J. F. "You Can Teach Problem Solving." Arithmetic Teacher, 1977, 25, 16-20.

Lesh, R. A. and Mierkiewicz, D. B. (eds.) Recent research concerning the development of spatial and geometric concepts. Columbus, Ohio: ERIC/SMEAC, 1977.

Lester, F. K., Jr. "Ideas about Problem Solving: A Look at Some Psychological Research." Arithmetic Teacher, 1977, 25, 12-14.

Levy, J., Trevarthen, C., and Sperry, R. W. "Perception of Bilateral Chimeric Figures Following Hemispheric Deconnexion." Brain, 1972, 95, 61-78.

McGlone, J. and Davidson, W. "The Relationship between Cerebral Speech Laterality and Spatial Ability with Special Reference to Sex and Hand Preference." Neuropsychologia, 1973, 11, 105-113.

McKim, R. H. Experiences in Visual Thinking. Monterey, Calif.: Brooks-Cole Publ., Co., 1972.

Maccoby, E. E. and Jacklin, C. N. Sex Differences in Intellectual Functionings, in Anon. Assessment in a Pluralistic Society. Proceedings of the 1972 Invitational Conference on Testing Problems. Educational Testing Service, Princeton, N. J. 1973, 37-55.

Maier, N. R. F. Problem Solving and Creativity in Individuals and Groups. Belmont, Calif.: Brooks-Cole, 1970.

Michael, W. B., Guilford, J. P., Fruchter, B., and Zimmerman, W. S. "The Description of Spatial-Visualization Abilities." Educational and Psychological Measurement, 1957, 17, 185-199.

Michael, W. B., Zimmerman, W. S., and Guilford, J. P. "An Investigation of the Nature of the Spatial Relations and Visualization Factors in Two High School Samples." Educational and Psychological Measurement, 1951, 11, 561-577.

Moyer, J. C. "The Relationship between the Mathematical Structure of Euclidean Transformations and the Spontaneously Developed Cognitive Structures of Young Children." Journal for Research in Mathematics Education, 1978, 9, 83-92.

Ornstein, R. E. The Psychology of Consciousness. San Francisco: W. H. Freeman & Co., 1972.

Piaget, J. and Inhelder, B. The Psychology of the Child. New York: Basic Books, Inc., 1969.

Robinson, E. "On The Uniqueness of Problems in Mathematics." Arithmetic Teacher, 1977, 25, 22-26.

Romberg, T. A. and Wilson, J. W. "The Development of Tests for the National Longitudinal Study of Mathematics Abilities." Mathematics Teacher, 1968, 61, 489-495.

Samples, B. "Mind Cycles and Learning." Phi Delta Kappan, 1977, 58, 688-692.

Schultz, K. A. "Transformation Geometry Concepts in Elementary School Children." Paper presented at the National Council of Teachers of Mathematics Conference, San Diego, Calif., April 14, 1978.

Schultz, K. A. Variables Influencing the Difficulty of Rigid Transformations during the Transition between the Concrete and Formal Operational Stages of Cognitive Development (Doctoral Dissertation, Northwestern University, 1977) Dissertation Abstracts International, 1978, 38, 5313A.

Sheehan, T. J. "Patterns of Sex Differences in Learning Mathematical Problem-Solving." Journal of Experimental Education, 1968, 36, 86.

Simon, H. A. Learning with Understanding. Columbus, Ohio: ERIC Information Analysis Center for Science, Mathematics, and Environmental Education, 1976. (ERIC No. ED 113206).

Suydam, M. N. and Weaver, J. F. "Research on Problem Solving: Implications for Elementary School Classrooms." Arithmetic Teacher, 1977, 25, 40-42.

Werdelin, I. Geometrical Ability and the Space Factors. Lund, Copenhagen, 1961.

Wheatley, G. H. "The Right Hemisphere's Role in Problem Solving." Arithmetic Teacher, 1977, 25, 36-69.

Wheatley, G. H., Kraft, R., Frankland, R. L., and Mitchell, R. "Hemispheric Specialization and Cognitive Development: Implications for Mathematics Education." Journal of Research in Mathematics Education, 1978, 9, 20-32.